University of Ottawa
School of Information Technology and Engineering

CSI 3104, Winter 2007

Assignment #1 Due date: Wednesday, January 19, 2007 at 8pm

IMPORTANT:
Each student is required to do each assignment individually. Late assignments will NOT be accepted; they will receive a grade of 0.
An e-mail, with your assignment attached, should be sent to the corrector (TBD) by the due date.
Moreover, deposit a hard copy of your assignment in the LOCKED box marked CSI 3104 at the drop off area located at the south end of the first floor of the SITE building. You must staple the pages of your assignment together, and put it in an envelope. Include your Name, Student Number, Course number, and Assignment number on every page as well as on the envelope.
1. Consider the language PALINDROME over the alphabet \( \{a, b\} \). It contains words that are same as their reverse words.

a) Consider the following recursive definition of PALINDROME:
   Rule 1. \( a \) and \( b \) are in PALINDROME.
   Rule 2. If \( x \) is in PALINDROME then so are \( axa \) and \( bxb \).

   Unfortunately the words defined by the rules have odd lengths. Fix the problem such that all appropriate words are included.

b) Prove that if \( x \) is in PALINDROME then so is \( x^n \) for any \( n \).

   c) Prove that if \( z^n \) is in PALINDROME (for positive integer \( n \)) then so is \( z \).

   d) Prove that PALINDROME has as many words of length \( 2n \) as it does of length \( 2n-1 \). How many words is that?

2. A language \( L \) is smaller than another language \( L \) if \( L \subset L \) and \( L \neq L \). Let \( T \) be any language closed under concatenation; that is, if \( t_1 \in T \) and \( t_2 \in T \) then \( t_1t_2 \) is also an element of \( T \). Show that if \( T \) contains \( S \) but \( T \neq S^* \), then \( S^* \) is smaller than \( T \). That is, \( S^* \) is the smallest closed language containing \( S \).

3. Give recursive definition for the following languages over \( \{a, b\} \):

   a) AA containing all words containing the substring \( aa \).

   b) NOTAA containing all words not containing the substring \( aa \).

4. Construct a regular expression defining each of the following languages over the alphabet \( \{a, b\} \).

   a) All strings such that the number of \( a \)'s is a multiple of 3.

   b) All strings such that the number of \( a \)'s is odd.

5. Construct a regular expression over \( \{a, b\} \) of all words that do not have both the substrings \( bba \) and \( abb \).

6. Construct a regular expression over \( \{a, b\} \) containing all strings that have an even number of \( a \)'s and an odd number of \( b \)'s.

7. Show that the following pair of regular expressions define the same language over alphabet \( \{a, b\} \):

   \[ a(ba+a)^*b \quad \text{and} \quad aa^*b(aa^*b)^* \text{.} \]

8. Show that the following pair of regular expressions define the same language over alphabet \( \{a, b\} \):

   \[ a(aa)^*(\Lambda+a)b + b \quad \text{and} \quad a^*b \text{.} \]