1. a) Rule 1: $\Lambda, a, b$ are in PALINDROME
   Rule 2: Ok

1. b) To determine if a string is PALINDROME:
   Step 1: If length (string) < 2 then the string is PALINDROME, Otherwise, continue.
   Step 2: Compare the first letter(s) with the reverse of the last letter(s). If they match then delete them both and repeat Step 1.

Since $x$ is PALINDROME, $x = \text{reverse}(x)$. Following the above algorithm to test $x^n$, two copies of $x$ are repeatedly deleted (one from each, because they match) until the string is reduced either to $\Lambda$ (when $n$ is even) or to $x$ (when $n$ is odd). Both of which are PALINDROME, therefore $x^n$ is PALINDROME.

1. c) Continuing the proofs above applied now to the PALINDROME string $z^n$, repeatedly remove two copies of $z$ at a time, one from either end, until if $n$ is odd only a PALINDROME string $z$ remains. If $n$ is even, then stop shrinking the string when $zz$ remains. Note that any PALINDROME can be viewed as a string concatenated with its own reverse. $zz$ is PALINDROME and $zz = z \text{reverse}(z)$, implies that $z = \text{reverse}(z)$ and $z$ is PALINDROME.

1. d) By using the algorithm in 1. (a), we can reduce any PALINDROME to a central core of one or two letters. On $\{a, b\}$, there are as many PALINDROMES of length 2 ($aa, bb$) as there are of length 1 ($a, b$). To make PALINDROMES of length $2n$, choose a core of length 2, and then make $n$-1 choices for the letters to the left which determine the letters to the right. To make PALINDROMES of length $2n-1$, choose a core of length 1, and then make $n$-1 choices for the other letters. In each case $n$ choices determine the word. Since there are two choices for letters, there are $2^n$ PALINDROME words of length $2n$ or $2n-1$. 
2. Since $T$ is closed and $S \subseteq T$, any factors in $S$ concatenated together two at a time will be a word in $T$. Likewise, concatenating factor in $S$ any number of times produces a word in $T$. That is any word in $S^*$ is also in $T$. However we are given that $T \neq S^*$ so $T$ contains some words that are not in $S^*$. We can conclude that $S^*$ is a proper subset of $T$, in other words $S^*$ is smaller than $T$, and in symbols $S^* \subset T$.

3. a) $aa \in AA$
   If $x \in AA$, then $ax, xa, bx$ and $xb$ are in $AA$.

3. b) $a \in NOTAA, b \in NOTAA$
   If $x \in NOTAA$ then $bx$ and $xb \in NOTAA$
   If $x \in NOTAA$ and $x=by$ then $ax \in NOTAA$
   If $x \in NOTAA$ and $x=ya$ then $xa \in NOTAA$

4. a) $b^*(ab*ab*ab^*)^*$

4. b) $b*ab*(ab*ab^*)^*$

5. $(a + ba)*b^* + b^*a(a + ba)^*(\Lambda + b)$

6. EVEN-EVEN $(b + ab(bb)^a)$ EVEN-EVEN, where EVEN-EVEN stands for the regular expression $(aa + bb + (ab + ba)(aa + bb)^{(ab + ba)})^*$

7. Both regular expressions define words starting with $a$, ending with $b$ and having no $bb$ as substring.

8. $a(aa)^*$ gives all strings with an odd number of $a$'s. $(\Lambda+a)$ gives the option of even length strings of $a$'s. Every word must have one final $b$. $+b$ allows for the string with no $a$'s. This is the same as any number of $a$'s if any followed by a single $b$; $a^*b$. 