University of Ottawa **Winter 2005**
SITE

CSI 3104
SOLUTION MID-TERM EXAM
Professor: N. Zaguia

Question 1. [6 points]
Give a recursive definition for the set of strings of digits 0, 1, 2, 3, ...9 that cannot start with the digit 0 nor with the digit 1.
Solution
We call this set of strings POSITIVE01

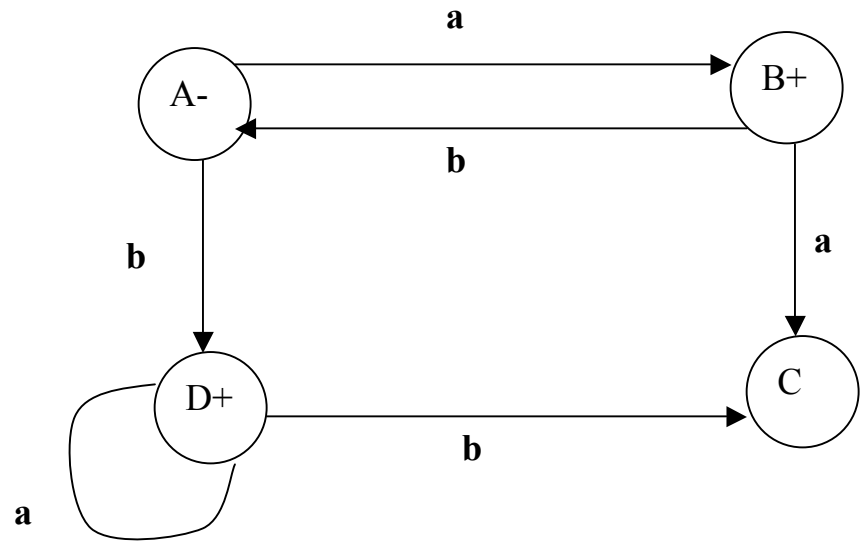
Rule 1: 2,3,4,5,6,7,8,9 are in POSITIVE01
Rule 2: If w is in POSITIVE01, w0, w1, w2, w3, w4, w5, w6, w7, w8, w9 are also words in POSITIVE01

Question 2. [6 points]
Show whether or not the following two regular expressions define the same language:

(a+b)*ba(a+b)* + ab* and (a+b) (a+b)*

Solution
(a+b)*ba(a+b)* + ab* corresponds to all words with "ba". A word without a "ba" should be a sequence as a's (may be empty) followed by a sequence of b's (may be empty) but not all of these are generated by ab*. For instance the word **aabb** is not generated by the expression (a+b)*ba(a+b)* + ab*.
However (a+b) (a+b)* corresponds to all non empty words on {a, b}. Therefore these two expressions don't define the same language.

Question 3. [8 points] YOU DON'T HAVE TO EXPLAIN YOUR ANSWERS.
We consider the following transition graph T.



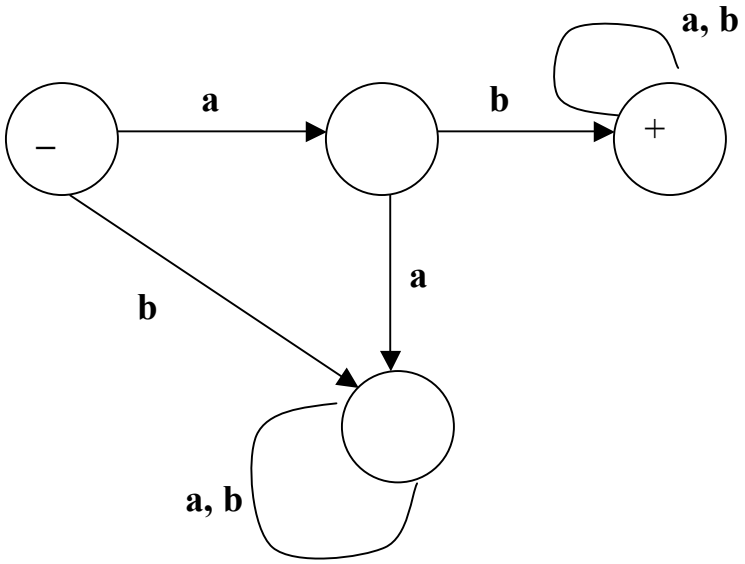
i) [4 points] Describe the language L accepted by T.
Solution
All words that consist of a sequence of ab's (may be empty) followed by a "b" and a sequence of a's (may be empty), as well as all words that start and end with an "a" and without any double letter.

ii) [4 points] Using the description found in (i), give a regular expression corresponding to the language L.
Solution

$$(ab)^*ba^* + a(ba)^*$$

Question 4. [10 points]
Let $\sum = \{a, b\}$ and let L be the language of all words on $\sum^*$ starting with ab.
Par instance, the word **abbbabb** is in L, however **aabbaba** is not in L.

(i) [4 points] Construct a finite automaton for the language L.
Solution



(ii) [2 points] Give a regular expression corresponding to the language L.
Solution

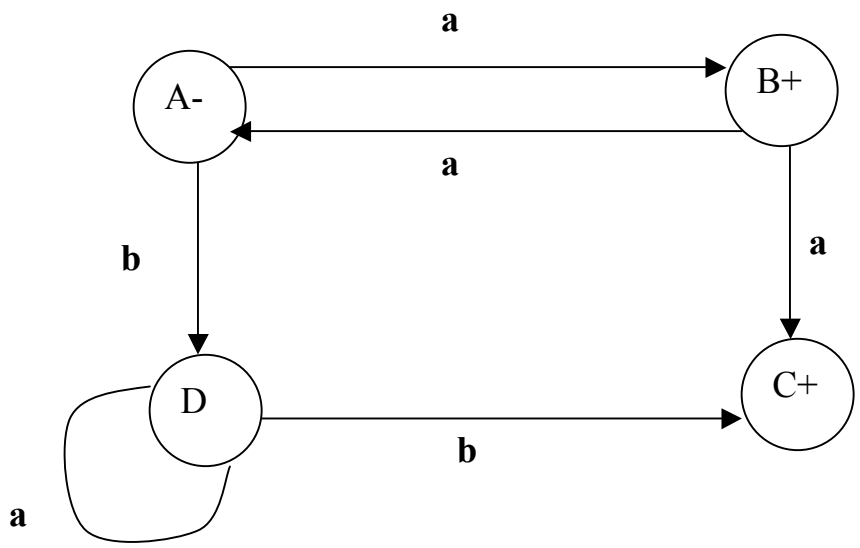
$$ab(a+b)^*$$

(iii) [4 points] Describe (in English phrases) the language L corresponding to the regular expression

$$E= (b + \Lambda)\ (ab)^*\ (a +\Lambda)$$

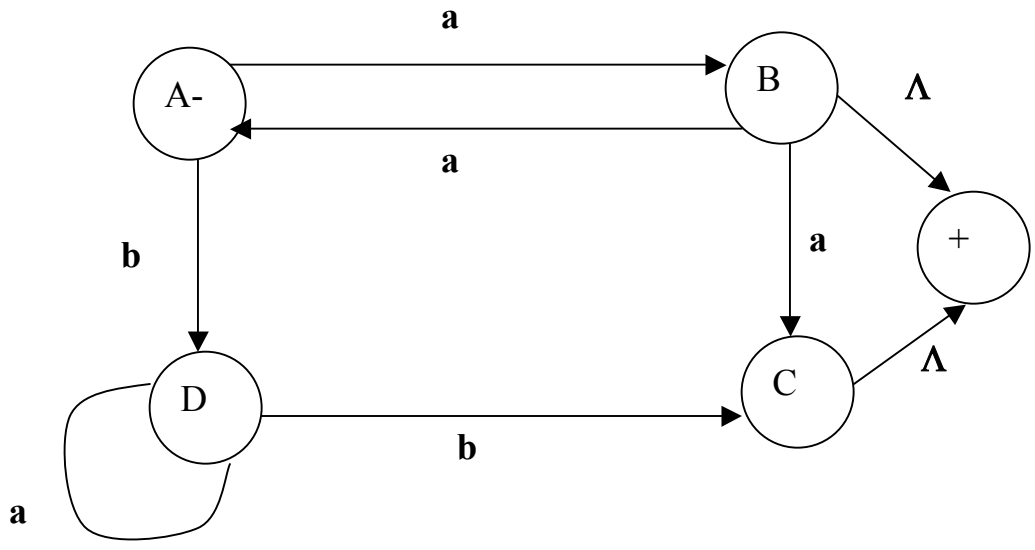
Solution
All words without two consecutive a's nor two consecutive b's.

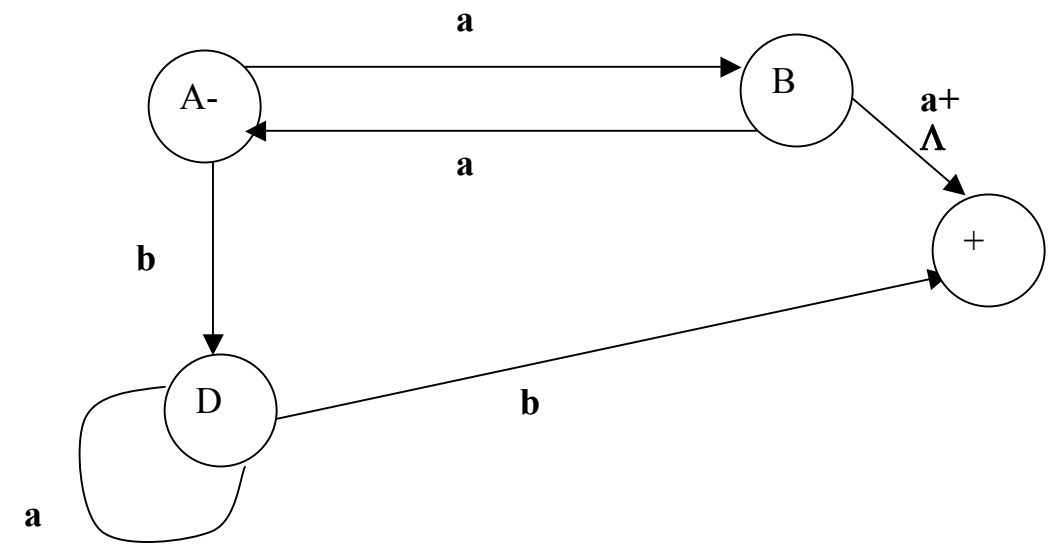
Question 5. [6 points] Using the method seen in class (Proof of Kleene's theorem), give a regular expression for the language accepted by the following transition graph:

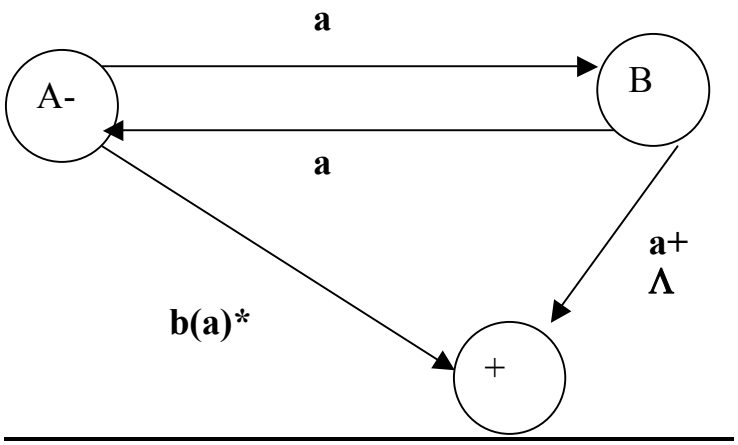


Solution

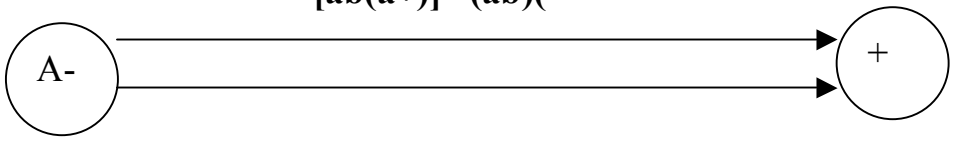
We start by creating a unique final state, then we delete the non final and initial states one by one.







[ab(a+)]* (ab)(



[ab(a+)]*

The regular expression is:

[ab(a+)]* (ab)(a+Λ) + [ab(a+)]* b(a)*b

Question 5. [4 points]
Let L be a regular language on $\sum = \{a, b\}$. Give an algorithm that will transform a transition graph for the language L into a new transition graph for the language complement of L. [EXPLAIN YOUR ANSWER]

[The complement L' of a language L is L'= $\sum^*$ - L]

Solution:

Algorithm:
First you should transform the transition graph into a finite automaton (in two steps: from Transition graph to a regular expression and then from the regular expression to a finite automaton. Both steps are a part of the proof of Kleen's theorem).
Then, we change in the finite automaton
final states into non final states and
non final states into final states.