1. Show the language $L=\{ a^{i+2000n},\ \ n \geq 0,\ 1 \leq i \leq 100\}$ as a regular expression. 13

$$a(a+\Lambda)^{99}(a^{2000})^{*}$$

$$(a+aa+\dots+\underbrace{aa\dots a}_{100})(\underbrace{aa\dots a}_{2000})^{*}$$

2. Is $(a+b)^{*} = (a^{*}b^{*})^{*}$? Prove that they are equal or find an example showing that they are different. 12

$(a^{*}b^{*})^{*} \subseteq (a+b)^{*}$ = ALL WORDS

$a = a^{1}b^{0} \in a^{*}b^{*}$

$b = a^{0}b^{1} \in a^{*}b^{*}$

$a+b \in a^{*}b^{*}$

$(a+b)^{*} \subseteq (a^{*}b^{*})^{*}$

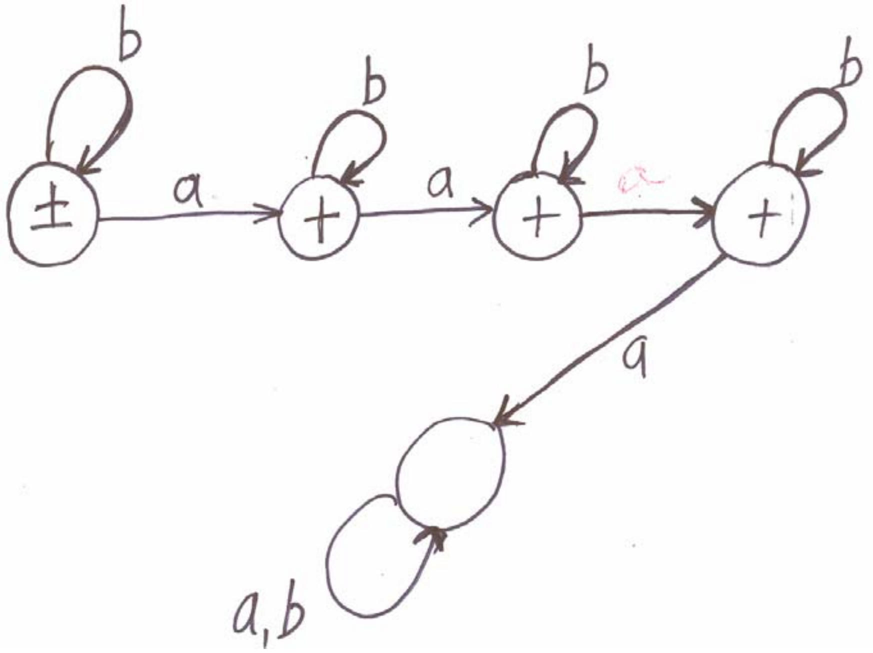
3. Find a regular expression (over alphabet $\{a, b\}$) for the language of all words with at most three a's in it.

12

$$b^*(a+\Lambda)\,b^*(a+\Lambda)\,b^*(a+\Lambda)\,b^*$$

4. Construct a finite automaton that accepts the language described in question 3. Do not use more than ten states.

13



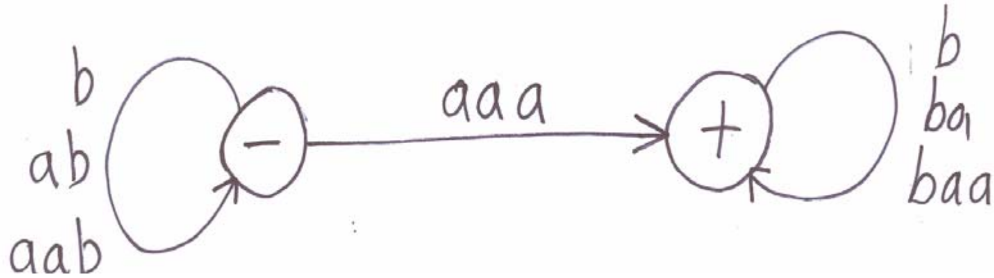
5. Find a regular expression for the language of all words where *aaa* appears exactly ones. For example, *babbaabaaabb* is in the language while *abbaabbaa, baaabbaaa* and *bbaaaabaa* are not.

13

$$(b+ab+aab)^*aaa(b+ba+baa)^*$$

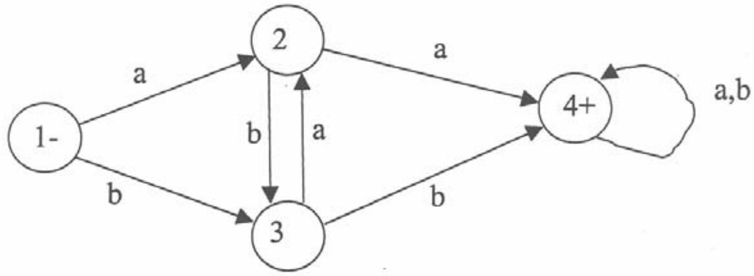
6. Find a transition graph which accepts the language described in question 5. Do not use more than 12 states.

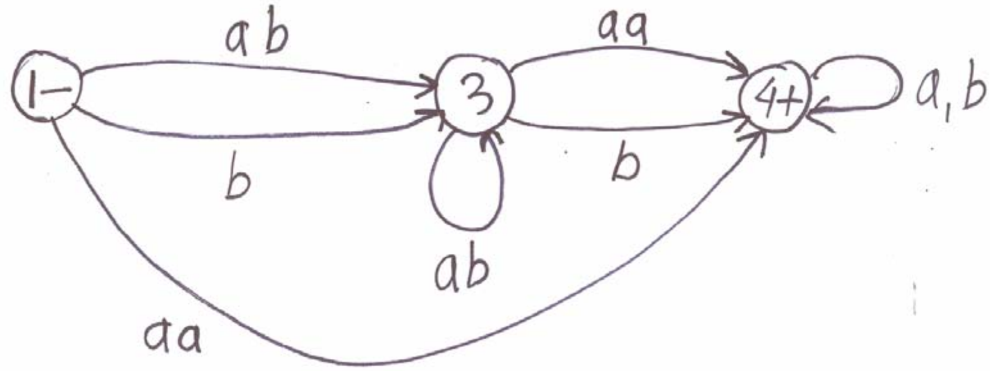
13

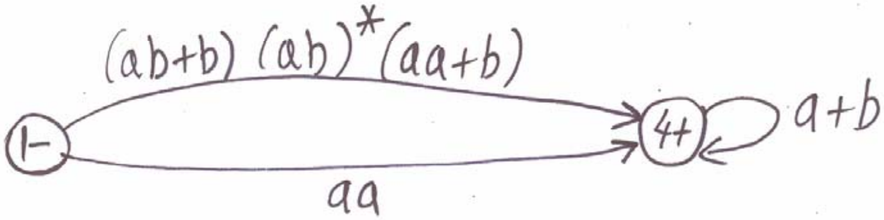


12

7. Following the procedure described in class and in the textbook (bypassing states), construct a regular expression that corresponds to the following finite automaton. Show all the steps in the process.







$[(ab+b)(ab)^*(aa+b)+aa](a+b)^*$

8. Let $\mathbf{r_1}$ and $\mathbf{r_2}$ be the following two finite automata. Following the construction for the product of two languages, find a finite automaton for the expression $\mathbf{r_1 r_2}$. 12